Math 60 7.4 Finding LCD and Forming Equivalent Rational Expressions

Objectives:

1) Find the Least Common Denominator (LCD) of two rational expressions
   a. Factor each denominator completely, including any negative GCF
   b. LCD is the product of each prime factor the greatest number of times from any denominator.
2) Write a rational expression that is equivalent to a given rational expression
   a. Determine which factor(s) is(are) missing from the given fraction
   b. Multiply the given fraction by 1, written as (missing factor)/ (missing factor).
   c. The correct answer is NOT in lowest terms or simplified.
3) Use the LCD to write equivalent rational expressions.
   a. Find the LCD (objective #1)
   b. Write equivalent rational expressions (objective #2)

Examples:

Find the LCD of the given pairs of fractions.

1) $\frac{5}{12}$ and $\frac{8}{15}$

2) $\frac{1}{6x^2y}$ and $\frac{2}{9xy^3}$

3) $\frac{5}{2a}$ and $\frac{7}{4a+4}$

4) $\frac{3b}{7a}$ and $\frac{2a}{7a+14b}$

5) $\frac{5}{x^2-x-6}$ and $\frac{9}{x^2+3x+2}$

6) $\frac{n+4}{n^2-9}$ and $\frac{8}{21-7n}$

7) Write $\frac{8}{3}$ as an equivalent fraction with denominator 12.

8) Use your calculator to show that your answers in the previous question are equal.

9) Write $\frac{7}{3x^2+2x}$ as an equivalent fraction with denominator $6x^2+4x$.

10) Show that your answers in the previous question are equivalent by evaluating them for $x=2$

Find the LCD of the given expressions and rewrite each rational expression with the LCD.

11) $\frac{3}{x^2-x-6}$ and $\frac{5}{x^2-9}$

12) $\frac{n+4}{n^2-9}$ and $\frac{8}{21-7n}$

Practice:

Find the LCD of the given expressions and rewrite each rational expression with the LCD.
(Use the LCD you found in the first examples.)

13) $\frac{1}{6x^2y}$ and $\frac{2}{9xy^3}$

15) $\frac{3b}{7a}$ and $\frac{2a}{7a+14b}$

14) $\frac{5}{2a}$ and $\frac{7}{4a+4}$

16) $\frac{5}{x^2-x-6}$ and $\frac{9}{x^2+3x+2}$

Find the LCD of the given expressions and rewrite each rational expression with the LCD.

17) $\frac{4n}{n^2-n-6}$ and $\frac{2}{n^2+4n+4}$

20) $\frac{4}{m^2-5m-6}$ and $\frac{2m}{m^2-12m+36}$

18) $\frac{4a}{a^2-1}$ and $\frac{7}{4a+4}$

21) $\frac{x+2}{x}$ and $\frac{x}{x+2}$

19) $\frac{2a}{a-2}$ and $\frac{a}{4-a^2}$

22) $\frac{2p^2-1}{6p}$ and $\frac{3p^3+2}{8p+8}$

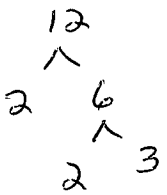
Math 60 SSM/3 7.4 Finding LCD and forming equivalent Rational Expressions

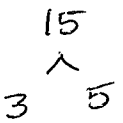
Objectives:
1) Find the LCD of two or more rational expressions
2) Write a rational expression that is equivalent to a given rational expression.
3) Use LCD to write equivalent rational expressions.

Find LCD.

(1) $\frac{5}{12}$ and $\frac{3}{15}$

Step 1: Factor each denominator completely





$12 = 2 \cdot 2 \cdot 3$

Step 2: Write the LCD as the product of the greatest number of copies of a factor:

2 : appears twice (2) in 12, 0 times in 15 (2) → 0
3 : appears once (1) in 12, once in 15 (1) → 0
5 : appears 0 times in 12, once (1) in 15 → 0 (1)

$LCD = 2 \cdot 2 \cdot 3 \cdot 5$

Step 3: (For numbers, but not expressions) Multiply

$LCD = 2 \cdot 2 \cdot 3 \cdot 5 = \boxed{60}$

(2) $\frac{1}{6x^2y}$ and $\frac{2}{9xy^3}$

$6x^2y = 2 \cdot 3 \cdot x \cdot x \cdot y$

$9xy^3 = 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$

MGO 7.4

factor	$6x^2y$	$9xy^3$
2	(1)	0
3	1	(2)
x	(2)	1
y	1	(3)

circle the largest number on the row

Use circled number of factors

$LCD = \underbrace{2}_{1} \cdot \underbrace{3 \cdot 3}_{2} \cdot \underbrace{x \cdot x}_{2} \cdot \underbrace{y \cdot y \cdot y}_{3}$

$$\boxed{LCD = 18x^2y^3}$$

Note: The numerators don't matter when finding LCD.

(3) $\frac{5}{2a}$ and $\frac{7}{4a+4}$

$2a = 2 \cdot a$

$4a+4 = 4(a+1) = 2 \cdot 2 (a+1)$

	$2a$	$4a+4$
2	1	(2)
a	(1)	0
$(a+1)$	0	(1)

Remember: (a) and (a+1) are different numbers, so they are different factors

$LCD = 2 \cdot 2 \cdot a \cdot (a+1)$

$$\boxed{LCD = 4a(a+1)}$$

Optional to show table

(4) $\frac{3b}{7a}$ and $\frac{2a}{7a+14b}$

$7a = 7 \cdot a$

$7a + 14b = 7 \cdot (a+2b)$

	$7a$	$7a+14b$
7	(1) ← tie →	1
a	(1)	0
$(a+2b)$	0	(1)

$$\boxed{LCD = 7a(a+2b)}$$

Always show your work for factoring

MGO 7.4

⑤ $\frac{5}{x^2-x-6}$ and $\frac{9}{x^2+3x+2}$

x^2-x-6 (−6; −3, 2; −1)
$= (x-3)(x+2)$

x^2+3x+2 (2; 2, 1; 3)
$= (x+1)(x+2)$

	x^2-x-6	x^2+3x+2
$(x-3)$	(1)	0
$(x+2)$	(1) ← tie →	1
$(x+1)$	0	(1)

$\boxed{LCD = (x+1)(x+2)(x-3)}$ ← Leave final answer factored

⑥ $\frac{n+4}{n^2-9}$ and $\frac{8}{21-7n}$

$n^2-9 = (n+3)(n-3)$

$21-7n = -7n+21$ standard form ← This step is essential!

$= -7(n-3)$ factor out negative GCF — Notice that the remaining factor is in the correct order.

	n^2-9	$21-7n$
$(n+3)$	(1)	0
$(n-3)$	(1) ← tie →	1
-7	0	(1)

$\boxed{LCD = -7(n+3)(n-3)}$

⑦ Write $\frac{8}{3}$ with denominator 12.

step 1: Factor desired denominator

12
2 ^ 6
2 ^ 3

$12 = 2 \cdot 2 \cdot 3$

step 2: Identify missing factors from denominator

$3 \rightarrow$ need $2 \cdot 2 \rightarrow 2 \cdot 2 \cdot 3$

Step 3: multiply by $1 = \frac{\text{missing factors}}{\text{missing factors}}$ from denom-inator

$$\frac{8}{3} \cdot \frac{1}{1}$$

$$= \frac{8}{3} \cdot \frac{2 \cdot 2}{2 \cdot 2} \quad \text{or} \quad \frac{3}{3} \cdot \frac{4}{4}$$

$$= \boxed{\frac{32}{12}}$$

Note: An equivalent fraction is NOT in lowest terms.

(8) $\frac{8}{3} = \boxed{2.\overline{6}}$

$\frac{32}{12} = \boxed{2.\overline{6}}$

equivalent fractions have equal value!

(9) Write $\frac{7}{3x^2+2x}$ as equivalent fraction with denominator $6x^2+4x$

Step 1: Factor both denominators

$3x^2+2x = x(3x+2)$

$6x^2+4x = 2x(3x+2)$

Step 2: Identify missing factor(s)

$x(3x+2) \longrightarrow$ need 2 $\longrightarrow 2x(3x+2)$

Step 3: Multiply by $1 = \frac{\text{missing factors}}{\text{missing factors}}$

$$\frac{7}{x(3x+2)} \cdot \frac{2}{2}$$

$$= \boxed{\frac{14}{2x(3x+2)}}$$

Notice: It's not simplified.

(10) Evaluate answers for $x=2$ to show they're equal.

$$\frac{7}{3x^2+2x} \longrightarrow \frac{7}{3(2)^2+2(2)} = \frac{7}{16} = \boxed{.4375}$$

cont →

m60 7.4

$$\frac{14}{2x(3x+2)} \rightarrow \frac{14}{2(2)(3(2)+2)} = \frac{14}{32} = \boxed{.4375}$$

Find LCD and rewrite equivalent fractions.

(11) $\frac{3}{x^2-x-6}$ and $\frac{5}{x^2-9}$

x^2-x-6 : -6, -3, $+2$, -1

$(x-3)(x+2)$

	x^2-x-6		x^2-9
$(x-3)$	(1)	← tie →	1
$(x+2)$	(1)		0
$(x+3)$	0		(1)

$x^2-9 = (x-3)(x+3)$

LCD $= (x-3)(x+2)(x+3)$

$$\frac{3}{x^2-x-6} = \frac{3}{(x-3)(x+2)} \cdot \frac{(x+3)}{(x+3)} = \boxed{\frac{3x+9}{(x-3)(x+2)(x+3)}}$$

↑ missing

$$\frac{5}{x^2-9} = \frac{5}{(x+3)(x-3)} \cdot \frac{(x+2)}{(x+2)} = \boxed{\frac{5x+10}{(x-3)(x+2)(x+3)}}$$

↑ missing

Distribute/FOIL numerators
BUT
Leave denominators factored

↳ This is because we'll add or subtract these fractions next, where we need to combine like terms in numerator, but leave denominator factored.

MGO 7.4

(12) $\frac{n+4}{n^2-9}$ and $\frac{8}{21-7n}$

wide awake: notice this is the same as #6!

LCD = $-7(n+3)(n-3)$

sound asleep: find the LCD first and do it all again!

$$\frac{n+4}{n^2-9} = \frac{n+4}{(n-3)(n+3)} \cdot \frac{(-7)}{(-7)} = \boxed{\frac{-7n-28}{-7(n-3)(n+3)}}$$

$$\frac{8}{21-7n} = \frac{8}{-7(n-3)} \cdot \frac{(n+3)}{(n+3)} = \boxed{\frac{8n+24}{-7(n-3)(n+3)}}$$

Alternate Answer: Move negative to numerator of second fraction when finding LCD.

$$\frac{n+4}{n^2-9} = \frac{n+4}{(n+3)(n-3)}$$ same as before

$$\frac{8}{21-7n} = \frac{8}{-7(n-3)} = \frac{-8}{7(n-3)}$$ ← move negative up!

LCD = $7(n-3)(n+3)$

$$\frac{n+4}{n^2-9} = \frac{n+4}{(n+3)(n-3)} \cdot \frac{7}{7} = \boxed{\frac{7n+28}{7(n-3)(n+3)}}$$

$$\frac{8}{21-7n} = \frac{-8}{7(n-3)} \cdot \frac{(n+3)}{(n+3)} = \boxed{\frac{-8n-24}{7(n-3)(n+3)}}$$

This pair of answers is also correct! (And usually preferred.)

Practice

(13) $\frac{1}{6x^2y}$ and $\frac{2}{9xy^3}$ same as #2, LCD = $18x^2y^3$

$$\frac{1}{6x^2y} \cdot \frac{3y^2}{3y^2} = \boxed{\frac{3y^2}{18x^2y^3}}$$

$$\frac{2}{9xy^3} \cdot \frac{2x}{2x} = \boxed{\frac{4x}{18x^2y^3}}$$

m60 7.4

(14) $\frac{5}{2a}$ and $\frac{7}{4a+4}$ same as #3, LCD = $a(a+1)$

$\frac{5}{2a} \cdot \frac{2(a+1)}{2(a+1)} = \boxed{\frac{10a+10}{4a(a+1)}}$ ← $5 \cdot 2(a+1) = 10(a+1)$ $= 10a+10$

$\frac{7}{4a+4} = \frac{7}{4(a+1)} \cdot \frac{a}{a} = \boxed{\frac{7a}{4a(a+1)}}$

(15) $\frac{3b}{7a}$ and $\frac{2a}{7a+14b}$ same as #4, LCD = $7a(a+2b)$

$\frac{3b}{7a} \cdot \frac{(a+2b)}{(a+2b)} = \boxed{\frac{3ab+6b^2}{7a(a+2b)}}$

$\frac{2a}{7a+14b} = \frac{2a}{7(a+2b)} \cdot \frac{a}{a} = \boxed{\frac{2a^2}{7a(a+2b)}}$

↑ missing from denominator (a in the numerator doesn't count)

(16) $\frac{5}{x^2-x-6}$ and $\frac{9}{x^2+3x+2}$ same as #5, LCD = $(x+1)(x+2)(x-3)$

$\frac{5}{x^2-x-6} = \frac{5}{(x-3)(x+2)} \cdot \frac{(x+1)}{(x+1)} = \boxed{\frac{5x+5}{(x+1)(x+2)(x-3)}}$

$\frac{9}{x^2+3x+2} = \frac{9}{(x+1)(x+2)} \cdot \frac{(x-3)}{(x-3)} = \boxed{\frac{9x-27}{(x+1)(x+2)(x-3)}}$

(17) $\frac{4n}{n^2-n-6}$ and $\frac{-2}{n^2+4n+4}$

Factor n^2-n-6 $\quad -3 \times 2 \quad (-6, -1)$

$(n-3)(n+2)$

Factor n^2+4n+4 $\quad 2 \times 2 \quad (4, 4)$

$= (n+2)(n+2)$

$= (n+2)^2$

	n^2-n-6	n^2+4n+4
$n-3$	(1)	0
$n+2$	1	(2)

cont →

m60 7.4

$LCD = (n-3)(n+2)(n+2)$

$$\frac{4n}{n^2-n-6} = \frac{4n}{(n-3)(n+2)} \cdot \frac{(n+2)}{(n+2)} = \boxed{\frac{4n^2+8n}{(n-3)(n+2)^2}}$$

$$\frac{2}{n^2+4n+4} = \frac{2}{(n+2)(n+2)} \cdot \frac{(n-3)}{(n-3)} = \boxed{\frac{2n-6}{(n-3)(n+2)^2}}$$

(18) $\frac{4a}{a^2-1}$ and $\frac{7}{4a+4}$

Factor $a^2-1 = (a+1)(a-1)$

Factor $4a+4 = 4(a+1)$

$LCD = 4(a+1)(a-1)$

	a^2-1		$4a+4$
$(a+1)$	(1)	← tie →	1
$(a-1)$	(1)		0
4	0		(1)

$$\frac{4a}{a^2-1} = \frac{4a}{(a+1)(a-1)} \cdot \frac{4}{4} = \boxed{\frac{16a}{4(a+1)(a-1)}}$$

$$\frac{7}{4a+4} = \frac{7}{4(a+1)} \cdot \frac{(a-1)}{(a-1)} = \boxed{\frac{7a-7}{4(a+1)(a-1)}}$$

(19) $\frac{2a}{a-2}$ and $\frac{a}{4-a^2}$

Factor $a-2 = (a-2)$ already factored

Factor $4-a^2 = -a^2+4$ standard form

$= -(a^2-4)$ GCF -1

$= -(a-2)(a+2)$

Option #1 $LCD = -(a-2)(a+2)$

$$\frac{2a}{a-2} \cdot \frac{-(a+2)}{-(a+2)} = \boxed{\frac{-2a^2-4a}{-(a-2)(a+2)}}$$

$$\frac{a}{4-a^2} = \boxed{\frac{a}{-(a-2)(a+2)}}$$ already has desired denominator

M60 7.4

Option #2 LCD = $(a-2)(a+2)$

$$\frac{2a}{a-2} \cdot \frac{(a+2)}{(a+2)} = \boxed{\frac{2a^2+4a}{(a-2)(a+2)}}$$

$$\frac{a}{4-a^2} = \frac{a}{-(a+2)(a-2)} = \boxed{\frac{-a}{(a+2)(a-2)}}$$

(20) $\frac{4}{m^2-5m-6}$ and $\frac{2m}{m^2-12m+36}$

Factor m^2-5m-6 (−6, +1, −5)
$= (m-6)(m+1)$

Factor $m^2-12m+36$ (36, −6, −6, −12)
$= (m-6)(m-6)$
$= (m-6)^2$

	m^2-5m-6	$m^2-12m+36$
$m-6$	1	(2)
$m+1$	(1)	0

LCD = $(m+1)(m-6)^2$

$$\frac{4}{m^2-5m-6} = \frac{4}{(m+1)(m-6)} \cdot \frac{(m-6)}{(m-6)} = \boxed{\frac{4m-24}{(m+1)(m-6)^2}}$$

$$\frac{2m}{m^2-12m+36} = \frac{2m}{(m-6)(m-6)} \cdot \frac{(m+1)}{(m+1)} = \boxed{\frac{2m^2+2m}{(m+1)(m-6)^2}}$$

(21) $\frac{x+2}{x}$ and $\frac{x}{x+2}$

Factor x
Factor $(x+2)$
} already factored

LCD = $x(x+2)$

	x	$x+2$
x	(1)	0
$x+2$	0	(1)

$$\frac{x+2}{x} \cdot \frac{x+2}{x+2} = \boxed{\frac{x^2+4x+4}{x(x+2)}}$$

$$\frac{x}{x+2} \cdot \frac{x}{x} = \boxed{\frac{x^2}{x(x+2)}}$$

m60 7.4

(22) $\frac{2p^2-1}{6p}$ and $\frac{3p^3+2}{8p+8}$

Factor $6p = 2 \cdot 3 \cdot p$

Factor $8p+8 = 2 \cdot 2 \cdot 2 \cdot (p+1)$

	$6p$	$8p+8$
2	1	(3)
3	(1)	0
p	(1)	0
$(p+1)$	0	(1)

$LCD = 2 \cdot 2 \cdot 2 \cdot 3 \cdot p(p+1)$

$= \boxed{24p(p+1)}$

$\frac{2p^2-1}{6p} \cdot \frac{4(p+1)}{4(p+1)} = \boxed{\frac{8p^3+8p^2-4p-4}{24p(p+1)}}$

$(2p^2-1)(4p+4)$

$= 8p^3+8p^2-4p-4$

$\frac{3p^3+2}{8p+8} \cdot \frac{3p}{3p} = \boxed{\frac{9p^4+6p}{24p(p+1)}}$